Calculating Uncertainty For the Analog Ohmmeter

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The Radio Shack VOM is an analog multimeter which can be used to measure voltage, current, and resistance. Determining the uncertainty of its measured values is simple enough — regardless of which function the meter is set for, the uncertainty of the measured value is based on the uncertainty in the needle position, which is reported by the manufacturer to be 3% of full deflection (except for AC voltage and the more sensitive DC voltage ranges, where the uncertainty is a little higher). This basically implies that the uncertainty in voltages and currents obtained by the VOM is 3% of the range setting for that measurement (meaning, for example, that a $15\,\mathrm{V}$ reading using the $50\,\mathrm{V}$ range setting yields an uncertainty of $(0.03)(50\,\mathrm{V}) = 1.5\,\mathrm{V}$ in the $15\,\mathrm{V}$ reading).

Calculation of the uncertainty of the voltage and current values obtained from the VOM is straightforward because the scales are linear — the voltage and current readings are proportional to the needle deflection. Unfortunately, the resistance readings are not. The green ohmmeter scale located on the top part of the analog display is non-linear. Thus, a 3% uncertainty in the needle position has to be translated to an uncertainty in the resistance value using a ruler. Since the entire green ohmmeter scale is $10\,\mathrm{cm}$ long, 3% full deflection corresponds to $3\,\mathrm{mm}$ to either side of the needle position. The uncertainty in the resistance has to be obtained by estimating the changes in the resistance reading when the needle physically moves $3\,\mathrm{mm}$ in either direction.

Determining the uncertainty in the VOM reading with a ruler is an extremely clumsy operation. Not only is it necessary to have a metric ruler on hand, but it is impossible to superimpose the ruler on the scale with any degree of precision because of parallax effects (the ruler is too far away from the scale). Surely there must be a better way of calculating the uncertainty for the VOM ohmmeter.

It turns out there is a better way. If an exact relationship between needle position and green ohmmeter scale reading can be worked out, then the 3% uncertainty in the needle deflection can be translated to an uncertainty in the resistance value using calculus. In fact, this relationship has now been worked out. It is

$$f = \frac{1}{1 + \rho/10}\,, \tag{1}$$

where f is the fractional needle deflection ($f = 1$ corresponds to full deflection), and ρ is the reading from the ohmmeter scale. To see that this relationship is reasonable, we first note that Eq. 1 correctly predicts that ρ decreases as f increases, which is plainly evident by examining the green ohmmeter scale. We can also verify certain special cases: $\rho = 0$ (which occurs at full deflection, $f = 1$), $\rho = 10$ (which occurs at precisely half deflection, $f = \frac{1}{2}$), $\rho = 30$ ($f = \frac{1}{4}$), $\rho = 40$ ($f = \frac{1}{5}$), and $\rho = \infty$ (zero deflection, $f = 0$). In each of these cases, Eq. 1 is correct.

The method used to determine Eq. 1 is a good illustration of experimental technique, and will be explained in the next section. For now, let us work out the consequences of this relationship on the uncertainty calculation. First of all, a relationship between f and ρ can be used to determine a relationship between δf and $\delta\rho$ by using the derivative rule,

$$\delta f = \left|\frac{df}{d\rho}\right| \delta\rho = \frac{1}{10}(1+\rho/10)^{-2}\delta\rho\,. \tag{2}$$

An uncertainty in the needle position of 3% full deflection implies that $\delta f = 0.03$. Substituting this into Eq. 2 and solving for $\delta\rho$ yields

$$\delta\rho = 10\delta f(1+\rho/10)^2 = 0.3(1+\rho/10)^2\,. \tag{3}$$

Note that ρ represents the ohmmeter scale reading itself. The actual resistance value is obtained by multiplying the reading by the resistance multiplier. We can write this as $R = \rho R_0$, where R is the actual resistance, ρ is the ohmmeter scale reading, and R_0 is the multiplier. Combining this with Eq. 3 yields the uncertainty for the resistance value itself,

$$\delta R = \delta\rho R_0 = 0.3(1+\rho/10)^2 R_0\,. \tag{4}$$

1. Suppose we set the ohmmeter on the "$R \times 10$" range (indicating a multiplier $R_0 = 10\,\Omega$) and measure the resistance of a resistor. The needle reads $\rho = 50$ on the green ohmmeter scale. This implies a resistance $R = \rho R_0 = (50)(10\,\Omega) = 500\,\Omega$ and an uncertainty $\delta R = 0.3(1+\rho/10)^2 R_0 = 0.3(1+50/10)^2(10\,\Omega) = 108\,\Omega$. Thus, $R = (500 \pm 110)\Omega$ (rounding the uncertainty to two significant figures).

2. Suppose we set the ohmmeter on the "$R \times 1\text{k}$" range ($R_0 = 1000\,\Omega$) and measure $\rho = 10$ on the green scale. Then the resistance is $10000\,\Omega$ and the uncertainty is $0.3(1+10/10)^2(1000\,\Omega) = 1200\,\Omega$, so $R = (10000 \pm 1200)\Omega$. It sure is nice not using that ruler.

3. Suppose we set the ohmmeter on the "$R \times 10\text{k}$" range ($R_0 = 10000\,\Omega$) and measure the same resistor as in Example 2. In this case the needle will read $\rho = 1.0$ and the uncertainty will be $0.3(1+1.0/10)^2(10000\,\Omega) = 3630\,\Omega$, so $R = (10000 \pm 3600)\,\Omega$. Clearly, the uncertainty depends on the range setting — it turns out that best results (lowest uncertainties) occur when the needle is near half deflection ($\rho = 10$), a result which is easily verified (see below). It behooves us to choose a range setting which brings the needle as close to the middle of the scale as possible.

There are times when it is more appropriate to calculate the relative uncertainty of the resistance. It is given by

$$\begin{aligned}\frac{\delta R}{R} = \frac{\delta\rho}{\rho} &= \frac{0.3(1+\rho/10)(1+\rho/10)}{\rho} \\ &= 0.3(1+\rho/10)(1/\rho+1/10) \\ &= 0.03(1+\rho/10)(1+10/\rho) \\ &= 0.03(2+\rho/10+10/\rho)\,.\end{aligned} \tag{5}$$

By setting the derivative of Eq. 5 to zero, it is possible to show that the minimum possible relative uncertainty from the analog ohmmeter occurs when the needle is deflected half way ($\rho = 10$), corresponding to a relative uncertainty of $0.03(2+1+1) = 0.12 = 12\%$. Thus, the analog ohmmeter has a 12% precision at best. Near either edge of the scale, the uncertainties can be quite large: at $\rho = 1$ or $\rho = 100$ the relative uncertainty is $0.03(2+10+0.1) = 36.3\%$.

How this relationship was determined

So, you are looking at the green ohmmeter scale on the VOM and you have just decided that you want to determine the relationship between the needle position and ohmmeter reading in algebraic form so that you can calculate uncertainties without having to resort to measuring the uncertainty range with a ruler. How do you go about it? In this section, we explain how Eq. 1 was derived.

Determining the algebraic form of a relationship between two quantities generally involves two steps: (1) making some sort of "educated guess" as to what the functional form of the relationship might look like (possibly with unknown parameters), and (2) taking enough data to solve for the unknown parameters and to make sure that the functional form really does describe the relationship. The second step is a simple matter of curve fitting — once you have a general functional form, there are computer programs that will do the fit for you. The first step, however, requires a bit of deductive reasoning and often a bit of ingenuity.

When approaching the problem of determining the functional relationship between the needle deflection and the resistance reading on the ohmmeter, the first question we should ask is how might the analog ohmmeter work. It is, after all, an *analog* meter, meaning that its functionality should be based on simple physical principles rather than solid state technology.

Fortunately, the operation of the analog ohmmeter is relatively straightforward and has been documented [J. Court, PHYC 4BL Meters Lab].[1] The needle deflects in response to a current passing through some sort of mechanism attached to the needle, with the needle deflection proportional to the current. This relationship can be expressed as $I_{\mathrm{mech}} = fI_0$ where I_{mech} is the current passing through the mechanism, f is the fractional needle deflection ($f = 1$ corresponds to full deflection), and I_0 is the amount of current needed for full deflection ($18\,\mu$A according to the given source above). The ammeter and the voltmeter can certainly take advantage of this property. To measure current, the VOM simply passes some fraction of that current through the needle mechanism. To measure voltage, the VOM simply places a large resistor in series with the needle mechanism and measures the current that is drawn from the circuit. Both circuits are shown in Figures 1(a) and 1(b), respectively. In both cases, the measured value is proportional to the current passing through the needle mechanism, and thus the ammeter/voltmeter scale on the VOM is a linear scale.

[1] Information found in this source was presumably obtained from manufacturer documentation and/or by taking the instrument apart and looking inside. It should be noted that reference is being made to J. Court's original "Meters Lab", and not the version currently in use in PHYC 4BL.

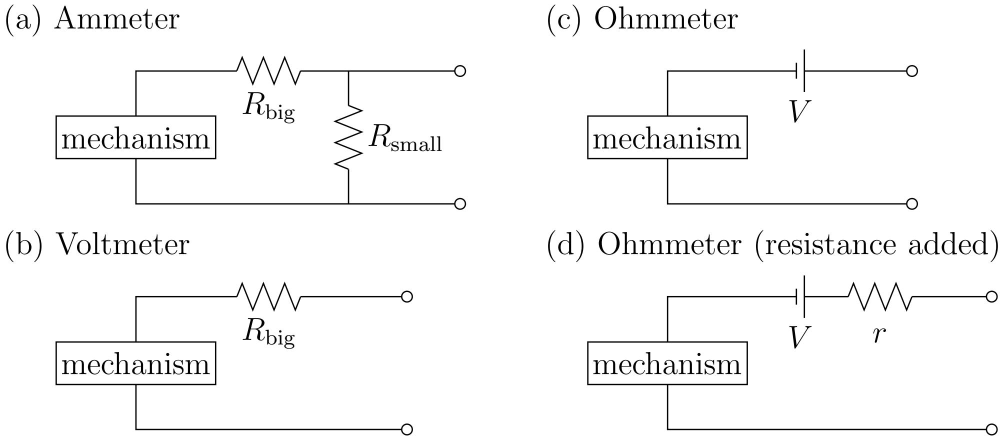

Figure 1: (a) Ammeter design ($I = I_{\text{mech}}(R_{\text{big}}/R_{\text{small}} + 1)$), (b) Voltmeter design ($V = I_{\text{mech}}R_{\text{big}}$), (c) Ohmmeter design, (d) Ohmmeter design (resistance added).

The ohmmeter is based on similar principles. When a resistor is connected to the ohmmeter, the ohmmeter applies a known voltage V to that resistor and then measures the current. Thus an ohmmeter might be designed as shown in Figure 1(c). However, there is one problem with this design. Suppose the ohmmeter is asked to measure a very small (essentially zero) resistance. In fact, it is very reasonable to use the ohmmeter this way — sometimes we use an ohmmeter to determine if two points in a (currently unpowered) circuit are directly connected or not. With the ohmmeter design in Figure 1(c), the result would be a very large current passing through the needle mechanism. In order to limit this current, we should place a resistance r in the ohmmeter which will be in series with the resistance we are trying to measure. That way, the total resistance in the circuit can never be less than r, no matter what resistance we are trying to measure. The ohmmeter design including this resistance is given in Figure 1(d).

Now suppose we connect a resistance R to the two free terminals of the ohmmeter depicted in Figure 1(d). Ohm's Law tells us that the resistance R is related to the current I_{mech} passing through the mechanism according to

$$V = (r + R)I_{\text{mech}}\,. \tag{6}$$

Using the fact that $I_{\text{mech}} = fI_0$ and that the ohmmeter scale is calibrated so that $R = \rho R_0$ where ρ is the ohmmeter scale reading and R_0 is the multiplier, we get

$$V = (r + \rho R_0)I_0 f\,.$$

Solving for f yields

$$f = \frac{V}{I_0(r + \rho R_0)} = \frac{1}{(I_0 r/V) + (I_0 R_0/V)\rho} = \frac{1}{a + b\rho}\,, \tag{7}$$

where $a = I_0 r/V$ and $b = I_0 R_0/V$ are parameters which depend on the values of I_0, r, V, and R_0.

f	ρ	f	ρ	f	ρ	f	ρ
1.000	0	0.845	1.8	0.685	4.5	0.340	20
0.980	0.2	0.833	2.0	0.665	5.0	0.300	24
0.960	0.4	0.820	2.2	0.625	6.0	0.250	30
0.940	0.6	0.800	2.4	0.593	7.0	0.200	40
0.920	0.8	0.793	2.6	0.553	8.0	0.160	50
0.905	1.0	0.775	2.8	0.522	9.0	0.090	100
0.890	1.2	0.765	3.0	0.500	10	0.040	300
0.875	1.4	0.740	3.5	0.455	12	0.020	500
0.860	1.6	0.710	4.0	0.400	15		

Table 1: Table of data obtained from the VOM ohmmeter scale.

The next step is to take enough data in order to determine if there exists values of a and b for which Eq. 7 determines the correct relationship between f and ρ, and if so, what those values of a and b are. First, let us check to make sure our proposed functional form makes sense. Examining the VOM ohmmeter scale, we notice that f and ρ are inversely related — larger needle deflections are labeled with smaller resistance values. This is correctly predicted by Eq. 7, and is also reasonable on physical grounds, since smaller resistances in a circuit do result in larger currents. We can also see that zero deflection ($f = 0$) is labeled with $\rho = \infty$, again correctly predicted by Eq. 7, and also reasonable on physical grounds because zero current implies infinite resistance.

Our analysis so far has led us to the conclusion that Eq. 7 is a good candidate for the general functional relationship between needle deflection and the green ohmmeter scale reading. It is consistent with a reasonable ohmmeter design and seems to agree qualitatively with our observations of the green ohmmeter scale. Our next task is to try to determine the parameters a and b and to verify that Eq. 7 indeed *does* accurately portray the relationship between f and ρ *quantitatively* with those parameter values.

Ultimately, we have to go to the VOM itself and determine the needle deflection f for various ohmmeter readings ρ. This can be accomplished by comparing the green ohmmeter scale with the black ammeter/voltmeter scale, since the latter scale is linear. This was accomplished by lining up the needle with various tick marks on the green ohmmeter scale and then reading both the green ohmmeter scale value and the corresponding black ammeter/voltmeter scale value. The needle was controlled by connecting the VOM, set up as a *voltmeter*, to a well-behaved power supply and dialing the voltage on the power supply. The fine-tune voltage control made it possible to control the VOM needle with a great deal of precision. The data is given in Table 1.

Determining the parameter values a and b and verifying the correctness of Eq. 7 involves a least-squares curve fit. Given that there are two parameters a and b, it makes sense to try to fit a straight line to the data. Unfortunately, the relationship between f and ρ predicted by Eq. 7 is not linear, but a linear relationship can be easily derived by rearranging the

equation,

$$\frac{1}{f} = a + b\rho \, .$$

It is evident that Eq. 7 predicts a linear relationship between $1/f$ and ρ, and so $1/f$ is plotted versus ρ on a graph and a straight line is fit to this graph. The plot (generated by Microsoft Excel) is attached at the end of this document. The last two data points in Table 1 were omitted from the plot because the uncertainty in $1/f$ (which arises mainly from the limited precision in reading values of f) is very large when f is near zero ($\delta(1/f) = \delta f/f^2$). Unfortunately, Microsoft Excel does not properly handle uncertainties when performing a least-squares fit (it treats all data equally with no option to do otherwise).

The plot speaks for itself. All of the data lie very close to the line, indicating that Eq. 7 gives the correct functional form. The parameter values obtained from the plot are $a = 0.9943$ and $b = 0.1014$, which are very close to the values $a = 1$ and $b = 0.1$ implied by Eq. 1. In fact, we might be tempted to use our "more precise" values of a and b instead of Eq. 1. However, we should resist this temptation for two reasons:

1. The parameter values a and b were obtained from a least squares fit involving data which was obtained by reading the position of the needle on the black and green scales. This data is fraught with uncertainty, since it is unlikely that both the black and green scales will line up on a tick mark at the same time. In fact, on the several occasions where the two scales did line up perfectly ($(f, \rho) = (1, 0)$, $(1/2, 10)$, $(1/4, 30)$, and $(1/5, 40)$), Eq. 1 predicts these values exactly, whereas Eq. 7 with the "more precise" parameter values does not.

2. Suppose you were the manufacturer of the analog ohmmeter, and your job was to choose values for V and r (see below) and calibrate the ohmmeter scale. Would you pick values which lead to a complicated scale described by $a = 0.9943$ and $b = 0.1014$, or a simpler scale described by $a = 1$ and $b = 0.1$?

We close with one final comment about the design of the ohmmeter. Recall that $a = I_0 r/V$ and $b = I_0 R_0/V$, and therefore appear to depend on the resistance multiplier R_0, in addition to I_0, V, and r. However, a and b must be fixed by the calibration of the printed ohmmeter scale. How is it possible, then, to offer different range multipliers on the same ohmmeter? The answer lies in the values chosen for V and r (see Figure 1(d)). Solving for V and r as a function of the other variables yields $V = (1/b)I_0 R_0$ and $r = (a/b)R_0$. These values are adjusted when the different resistance multipliers are selected in order to maintain fixed values of a and b.[2] The different range settings of the voltmeter and ammeter are implemented in a similar manner. The values of R_{big} and R_{small} are adjusted according to the ammeter/voltmeter range selected.

[2]Given that R_0 varies by a factor of 10^4, it seems likely that I_0 is effectively adjusted as well as V and r. One way to accomplish this is to replace the needle mechanism in Figure 1(d) with the full ammeter depicted in Figure 1(a). Values of R_{big} and R_{small} can be adjusted in order to adjust the range of the ammeter, which represents the effective value of I_0.